On Modeling of Fluctuations in Quasi-Static Approach
Describing the Temporal Evolution of Retry Traffic

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Significant problems with the Internet include node failure due to congestion or overload.

One of the key factors behind overload is the generation of retry traffic.

We consider user impatience which is one of the cause of retry traffic.

**Retry traffic due to user impatience**

Users who can not endure their waiting time before starting the service might generate duplicate service requests.
An evaluation method that can accurately model retry traffic is important.

In previous work, we proposed the quasi-static approach that replicates the temporal evolution of these traffic.

- We separate a timescale of the transitions of user and system behavior in the approach.
- The quasi-static approach can evaluate an high-speed system in which we can not use simulation approach.

The aim of this study

In this study, we confirm the validity of the quasi-static approach by comparing the traditional Monte-Carlo simulations.
Quasi-static retry traffic model

- We focus on one of the simplest model as an example: M/M/1 with retry traffic.
- The rate of the retry traffic is depend on the queue length.

Long queue length bring high rate retry traffic, high rate retry traffic bring more long queue.
We modeled the behavior of retry traffic as the quasi-static retry traffic model.

Character of quasi-static retry traffic model

- The change of the queue is very faster compared with the user responses (We can separate the timescales).
- The change of the traffic rate is proportional to average queue length on past $T$ period.

Arrival rate is decided by average queue length on past $T$ period

Behavior of the queue length (the change of the queue is very faster)

Average queue length

$kT$, $(k+1)T$
Quasi-static approach

- In actual system, the traffic changes not discretely but continuously.

![Graph showing queue length over time](image)

- On finite speed system, an average queue length contains stochastic fluctuations.
- On infinite high speed system, the fluctuation is 0.

![Graph showing average queue length over time](image)
By considering the continuity and fluctuation, we describe the traffic behavior as the following Langevin equation:

\[
\frac{d}{dt}X(t) = F(X(t)) + \sqrt{D(X(t))} \xi(t)
\]

- **Behavior of the infinite high speed system**
- **Stochastic fluctuation**

\(X(t)\) : Number of arrivals in past \(T\) period
\(\xi(t)\) : White Gaussian noise

**Average behavior**
\[
F(X) = \lambda_0 - \frac{X}{T} + \varepsilon \frac{X/(\mu T)}{1 - X/(\mu T)}
\]

**Magnitude of the fluctuation**
\[
D(X) = \lambda_0 + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}
\]
It is well known that the Langevin equation is equivalent to the Fokker-Planck equation as shown by
\[
\frac{\partial}{\partial t} p(x, t) = -\frac{\partial}{\partial x} F(x)p(x, t) + \frac{1}{2} \frac{\partial^2}{\partial x^2} D(x)p(x, t)
\]

We can calculate the pdf \( p(x, t) \) of \( X(t) \), and can get the distribution.
Verification of the quasi-static approach

- To confirm the validity of our approach, we compare the result with that of Monte-Carlo simulations.

\[
F(X) = \lambda_0 - \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \quad D(X) = \lambda_0 + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}
\]

- The distributions that are computed by the Monte-Carlo and our approach are not corresponding.

Arrival rate without retry \( \lambda_0 \)
300

Service rate \( \mu \)
1000

Intensity of retry traffic \( \varepsilon \)
200

Users’ response time \( T \)
1 s

Simulation period \( t \)
50 s
Verification of the quasi-static approach (2)

- We reconsider $F(X(t))$ and $D(X(t))$.
  - $X(t)$ is a number of arrivals in past $T$ period.
  - Infinitesimal change $dX(t)/dt$ is a random variable.
  - $F(X(t))$ and $D(X(t))$ indicate mean and variance of $dX(t)/dt$.
- $dX(t)/dt$ is composed of increment and decrement of $X(t)$.

Mean and variance of each element are as follows.

Mean of increment : $\lambda_0$  
Variance of increment : $\lambda_0$  
Mean of decrement : $-X/T$  
Variance of decrement : $X/T$
Verification of the quasi-static approach (3)

We modified $F(X(t))$ and $D(X(t))$, and recalculate the distribution of the traffic.

\[
F(X) = \lambda_0 - \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \quad D(X) = \lambda_0 + \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}
\]

We can confirm that the quasi-static approach yields result similar to that of the Monte-Carlo simulation.
In this study, we verified the validity of quasi-static approach that describes the behavior of input traffic including retry traffic.

- We computed the temporal evolution of input traffic on a M/M/1 based system with retry traffic by using the quasi-static approach.
- We were able to confirm that the results are corresponding to that of Monte-Carlo simulations.
- Therefore, we confirmed that the quasi-static approach can appropriately evaluate a system with retry traffic.
Thank you very much for your kind attention.
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Quasi-static approach

Verification of the quasi-static approach

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